NASCA NQF LEVEL 4 DRAFT SUBJECT STATEMENT

MATHEMATICS

higher education
& training

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NQF LEVEL 4

DRAFT SUBJECT STATEMENT

MATHEMATICS

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NASCA CURRICULUM

MATHEMATICS (NQF LEVEL 4)

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INTRODUCTION

The National Senior Certificate for Adults (NASCA) aims to provide evidence that the adult learners are equipped with a sufficiently substantial basis of discipline-based knowledge, skills and values to enhance meaningful social, political and economic participation, to form a basis for further and/or more specialist learning and possibly to enhance the likelihood of employment. In these respects, the NASCA promotes the holistic development of adult learners.

SECTION 1

1.1 What is Mathematics?

Mathematics is a language that makes use of symbols and notations for describing numerical, geometric and graphical relationships. It is a human activity that involves observing, representing and investigating patterns and qualitative relationships in physical and social phenomena and between mathematical objects themselves. It helps to develop mental processes that enhance logical and critical thinking, accuracy and problem solving that will contribute in decision-making. Mathematical problem solving enables us to understand the world (physical, social and economic) around us, and, most of all, to teach us to think creatively.

1.2 Aims

The Mathematics curriculum is to provide students with the fundamental mathematical knowledge and skills.

The general aims of the mathematics curriculum are to empower learners to:

- Develop positive attitudes towards mathematics;
- Acquire the necessary mathematical knowledge, skills and values for continuous learning in mathematics and related disciplines, and for applications to the real world;
- Be able to understand and work with the number system
- Develop the necessary process skills for the acquisition and application of mathematical concepts and skills;
- Develop the mathematical thinking, problem solving and modelling skills and apply these skills to formulate and solve problems;
- Recognise and use connections among mathematical ideas, and between mathematics and other disciplines;
- Provide the opportunity to develop in learners the ability to be methodical, to generalise, make conjectures and try to justify or prove them;
- Make effective use of a variety of mathematical tools (including information and communication technology tools) in the learning and application of mathematics;
- Produce imaginative and creative work arising from mathematical ideas;
- Develop the abilities to reason logically, to communicate mathematically, and to learn cooperatively and independently;
- Collect, analyse and organise quantitative data to evaluate and comment on conclusions;
- To prepare the learners for further education and training as well as the world of work.

1.3 Specific Skills

To develop essential mathematical skills the learner should:

- Develop the correct use of the language of Mathematics;
- Collect, analyse and organise quantitative data to evaluate and critique conclusions;
- Use mathematical process skills to identify, investigate and solve problems creatively and critically;
- Use spatial skills and properties of shapes and objects to identify, pose and solve problems creatively and critically;
- Participate as responsible citizens in the life of local, national and global communities;
- Communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams.

1.4 Focus of Content Areas

Mathematics in the NASCA curriculum covers the following four main content areas:

1. Number and number relationships
2. Functions, Graphs and Algebra
3. Space, shape and measurement
4. Data handling

Each content area contributes towards the acquisition of the specific skills. The table below shows the main topics and sub-topics in the NASCA curriculum.

Content Topic	Sub- Topics
Number and number relationships	• Exponents and surds • Number patterns • Formulae to calculate the sum of Arithmetic and Geometric Sequences • Application of the knowledge of geometric series to solve problems • Investment and Loan options
Functions, Graphs and Algebra	• Manipulation and simplification of algebraic expressions • Solving of algebraic equations • Function concept • Various types of functions: straight line; parabola; hyperbola; exponential and trigonometric graphs • Characteristics of functions • Effects of parameters on graphs (manipulation of representations) • Inverses of functions: straight line; parabola and exponential graphs • Solving of cubic equations
Space, shape and measurement	• Measurement: Volume and surface area of geometric objects. • Conjecture, prove and use circle geometry theorems • Coordinate Geometry: distance between two points, gradient of a line, parallel and perpendicular lines, mid-point of a line segment, inclination of a line, equation of a line, equation of a circle and equation of a tangent to a circle • Trigonometry: Definition of the trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$; special angles; reduction formulae; specific and general solutions to trigonometric equations; sine, cosine and area rules; compound and double angles; problems in 2 and 3- dimensions
Data handling	• Univariate numerical ungrouped data: Measures of central tendency; five number summary; box and whisker diagrams; measures of dispersions • Univariate numerical grouped data: Frequency distribution table; Ogive curve; quartile values; histograms; mean, median and mode • Bivariate data: Variance and standard deviation of • Bivariate data: Scatter plots; Correlation; Best fit function; Regression lines

1.5 Weighting of sub-topic Content Areas

The weighting of mathematics content areas serves two primary purposes:

- Firstly, the weighing gives guidance on the amount of time needed to address the content adequately within each content area;
- Secondly, *the weighting gives* guidance on the spread of content in the examination (especially end of the year summative assessment).

Structure of the examination papers

Description	Duration	Weightings (Marks)
PAPER 1	**3h**	
Patterns, Sequences and Series		45 ± 3
Finance, growth and decay		35 ± 3
Functions, Graphs and Algebra		70 ± 3
TOTAL		**150**
PAPER 2:	**3h**	
Data handling		30 ± 3
Circle geometry & measurement		40 ± 3
Co-ordinate geometry		30 ± 3
Trigonometry		50 ± 3
TOTAL		**150**

1.6 Suggested study hours

Mathematics is a 30 credit course, which relates to 300 notional study hours.

It is envisaged that a typical one-year offering of the course will cover 30 weeks, excluding revision and examination time. Candidates should therefore spend 10 hours per week on mathematics. This should consist of 6 hours of face-to-face instruction and 4 hours of self-study.

A suggested time allocation for the course is shown on the table below:

Topic	**Face-to-face teaching time**	**Self-study time**
Number and number relationships	6 hours per week x 30 weeks	4 hours per week x 30 weeks
Functions, Graphs and Algebra		
Space, shape and measurement		
Data handling		
Total course hours	**300 notional hours**	

SECTION 2

2.1 EXIT LEVEL OUTCOMES

These exit level outcomes describe the knowledge, skills and abilities which candidates are expected to demonstrate at the end of the course. They reflect those aspects of the aims which will be assessed.

Exit Level Outcome 1:

Recognise, describe, represent and work with numbers and their relationships to estimate, calculate and check in solving problems.

Exit Level Outcome 2:

Investigate, analyse, describe and represent a wide range of functions and solve related problems.

Exit Level Outcome 3:

Describe, represent, analyse and explain properties of shapes in 2- and 3-dimensional space with justification.

Exit Level Outcome 4:

Collect and use data to establish statistical models to solve related problems.

2.2 ASSESSMENT CRITERIA PER EXIT LEVEL OUTCOME:

Exit Level Outcome 1: Assessment Criteria:

- Use laws of exponents and surds to simplify and solve equations
- Problems involving number patterns are identified and solved correctly.
  Range: Includes arithmetic and geometric sequences.
- Formulae are correctly proved and the sum of sequences are calculated correctly.
- Knowledge of geometric series is applied to solve problems correctly.
- Investment and loan options are critically analysed to make informed decisions as to the best option(s).

Exit Level Outcome 2: Assessment Criteria:

- Manipulation and simplification of algebraic expressions are done correctly.
- Algebraic equations are solved correctly
- A formal definition for the function concept is given correctly.
- Various types of functions and relations are worked with correctly.
- Characteristics of functions are correctly explained and graphs are correctly sketched.
- Situations are recognised in which it may be helpful to manipulate representations.
- Cubic equations are solved using the factor theorem and other techniques.

Exit Level Outcome 3: Assessment Criteria:

- Solve problems involving volume and surface area of geometric objects such cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms, right pyramids (with square, equilateral or hexagonal basis), right cones, spheres and combination of such geometrical objects.
- Investigate and prove theorems of the geometry of circles assuming results from earlier grades.

- Solve circle geometry problems, providing reasons for statements when required
- Prove circle geometry riders using circle geometry theorems and geometry results established in earlier grades.
- Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1; y_1)$and$(x_2; y_2)$, a formula for calculating:

  The distance between two points

  The gradient of the line segment joining the points

  Conditions for parallel and perpendicular lines

  The coordinates of the midpoints of the line segment joining the points.
- A two dimensional Cartesian co-ordinate system is used to derive and apply:

  The equation of a line through two given points

  The equation of a line through one point and parallel or perpendicular to a given line.

  The inclination of a line.

  The equation of a circle (any centre)

  The equation of tangent to a circle at a given point on the circle.
- Definitions of the trigonometric ratios $sin\theta$, $cos\theta$and $tan\theta$ in right angled triangles.
- Extend the definitions of $sin\theta$, $cos\theta$and $tan\theta$ to $0° \leq \theta \leq 360°$.
- Derive and use values of the trigonometric ratios for the special angles $\theta\epsilon\{0°; 30°; 60°; 90°\}$.
- Derive and use the identities $tan\theta = \frac{sin\theta}{Cos\theta}$ and $sin^2\theta + cos^2\theta = 1$.
- Derive and use the trigonometric reduction formulae.
- Determine the general solution and/or specific solutions of trigonometric equations.
- Establish and apply the sine, cosine and area rules.
- Prove and use of the compound angle and double angle identifies.
- Problems in 2- and 3-dimensions are solved by constructing and interpreting geometrical and trigonometric models

Exit Level Outcome 4: Assessment Criteria:

- Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical ungrouped data
- Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical grouped data
- Understand and demonstrate the use of variance and standard deviations as measures of dispersion of a set of bivariate data
- Understand the various concepts and representations relating to data handling and its uses, and used them correctly.

SECTION 3

SUBJECT CONTENT

The mathematical content for each topic/unit is detailed below. The order in which topics are listed is not intended to imply anything about the order in which they might be taught.

TOPIC 1: NUMBER and NUMBER RELATIONSHIPS

Overview of NUMBER and NUMBER RELATIONSHIPS
When solving problems, the learner is able to recognise, describe, represent and work confidently with numbers and their relationships to estimate, calculate and check solutions. In this topic learners should be able to: • Expand their capacity to represent numbers in a variety of ways and move flexibly between representations • Use laws of exponents and surds to simplify and solve equations • Calculate confidently and competently with and without a calculator, guarding against over-dependent on the calculator • Develop the concept of simple and compound growth decay • Solve problems related to arithmetic, geometric and other sequences and series including contextual problems related to hire-purchase, bond repayments and annuities. • Explore real-life and purely mathematical number patterns and problems which develop the ability to generalise, justify and prove.

Prior knowledge of the following sub topics and their extensions are required for this topic:

- Laws of exponents
- Operations with polynomials.
- Solving simultaneous equations
- Exchange rates

Details of Content coverage

1.1 Use laws of exponents and surds to simplify and solve equations

Content:

- Exponents and surds

Learners should be able to:

1.1.1 Simplify expressions and solve equations using the laws of exponents for rational exponents where

$x^{\frac{p}{q}} = \sqrt[q]{x^p}$; x> 0; q> 0

1.1.2 Add, subtract, multiply and divide simple surds.
1.1.3 Solve simple equations involving surds.

1.2 Investigate number patterns

Content:

- Linear and quadratic pattern
- First and Second constant difference

Learners should be able to:

1.2.1 Investigate number patterns leading to those where there is a constant difference between consecutive terms, and the general term is therefore linear

1.2.2 Investigate number patterns leading to those where there is a constant second difference between consecutive terms, and the general term is therefore quadratic

1.3 Identify and solve problems involving number patterns

Content

- Arithmetic Sequence
- Geometric Sequence
- Neither Arithmetic Sequence nor Geometric Sequence

Learning outcomes:

Learners should be able to:

1.3.1 Solve problems involving number patterns including but not limited to arithmetic and geometric sequences and series.

1.4 Prove formulae and use it to calculate the sum of sequences (or series) correctly.

Content:

- Sigma notation
- Formulae

Learning outcomes:

Learners should be able to:

1.4.1 Use and apply Sigma notation

1.4.2 Derive and apply the formulae for the sum of the series:

$$S_n = \frac{n}{2}\left[2a + (n-)d\right]$$

$$S_n = \frac{n}{2}(a + l)$$

$$S_n = \frac{a(r^n - 1)}{r - 1};\cancel{(r \neq 1)}$$

$$S_\infty = \frac{a}{1 - r};(-1 < r < 1),(r \neq 1)$$

1.5 Apply knowledge of geometric series to solve problems correctly.

Content:

- Bond Repayment
- Sinking Fund Problems

Learning outcomes:

Learners should be able to:

1.5.1 Apply knowledge of geometric series to solve annuity, bond repayment and sinking fund problems, with or without the use of the formulae:

$$F=\frac{x\left[(1+i)^n-1\right]}{i} \text{ and } P=\frac{x\left[1-(1+i)^{-n}\right]}{i}$$

1.5 Critically analyse investment and loan options to make informed decisions as to the best option(s).

Content:

- Simple and compound growth/decay formulae;
  - Interest, Hire Purchase and Inflation
  - Population Growth
  - Straight line and reducing balance Depreciation
- Annuity Formulae: Present and Future Values
  - Investment Options
  - Loan Options
- Time period
  - Logarithmic laws

Learning outcomes:

Learners should be able to:

1.6.1 Use the following logarithmic laws in the context of real life problems related to finance

$$\log_a(xy)=\log_a x+\log_a y$$

$$\log_a\left(\frac{x}{y}\right)=\log_a x-\log_a y$$

$$\log_a x^n = n\log_a x$$

$$\log_a b=\frac{\log b}{\log a}$$

Note: Manipulation of logarithmic laws will not be examined

1.6.2 Use simple and compound growth formulae:
A=P(1+in) and A=P(1+i)ⁿ to solve problems including Interest, hire Purchase, inflation and other real life problems.

1.6.3 Make use of logarithms to calculate the value of n, the time period in the equations:

$$A = P(1+i)^n or\ A = P(1-i)^n$$

1.6.4 Use the two annuity formulae to make decisions on which investment or loan option to choose:

$$F=\frac{x[(1+i)^n - 1]}{i} \text{ and } P=\frac{x[1-(1+i)^{-n}]}{i}$$

1.6.5 Use simple and compound decay formulae, *A=P (1-in) and A=P(1- i)*^{n} to solve problems including straight line depreciation and depreciation of a reducing balance.

TOPIC 2: FUNCTIONS, GRAPHS AND ALGEBRA

Overview of FUNCTIONS, GRAPHS AND ALGEBRA

In this section the learner should be able to investigate, analyse, describe and represent a wide range of functions and solve related problems.

A fundamental aspect of this outcome is that it provides learners with versatile and powerful tools for understanding their world while giving them access to the strength and beauty of mathematical structure. The language of algebra will be used as a tool to study the relationship between specific variables in a situation.

In this topic learners should be able to:

- Manipulate and simplify algebraic expressions
- Solve linear, quadratic, exponential and simultaneous equations
- Understand the concepts of a function and associated notations;
- Sketch and interpret various types of functions and relations, including linear, quadratic, exponential, rectangular hyperbola, trigonometric and some rational functions;
- Investigate the effect of changing parameters on the graphs of functions;
- Determine and sketch the inverse graphs of prescribed functions;
- Solve cubic equations using the factor theorem and other techniques.

Prior knowledge of the following topics and their extensions are required for this topic:

Functions and graphs

- Cartesian plane and coordinates
- Plotting a set of ordered number pairs
- Functions and relationships
- Linear relationships between two variables (linear functions).
- Generate straight line graphs by means of point-by-point plotting, intercept method and gradient-intercept method.
- Determining the equation of the straight line graph
- Gradient, x and y-intercept of a straight line
- Sketching the graph $y = x$
- Parallel and perpendicular lines

Algebra

- Solving equations by substitution
- Find products of a binomial with a binomial
- Simplify algebraic fractions with monomial denominators
- Exponential laws

Details of Content coverage

2.1 Manipulation and simplification of algebraic expressions are done correctly.

Content:

- Products of binomials with trinomials
- Factorisation
- Simplification of algebraic fractions

Learning outcomes:

Learners should be able to:

2.1.1 Find products of binomials with trinomials
2.1.2 Factorise by identifying/taking out of common factor.
2.1.3 Factorise by grouping in pairs
2.1.4 Factorise the difference of two squares.
2.1.5 Factorise trinomials.
2.1.6 Simplify algebraic fractions with binomial denominators.

2.2 Algebraic equations are solved correctly

Content:

- Solve:
  - linear equations
  - quadratic equations
  - exponential equations
  - simultaneous equations

Learning outcomes:

Learners should be able to:

2.2.1 Solve linear equations.
2.2.2 Solve quadratic equations by:
  - ➢ Factorisation
  - ➢ Using quadratic formula: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$
  - ➢ Completing the square

2.2.3 Solve exponential equations in the form ka^x (where x is an integer) by using the laws of exponents.
2.2.4 Solve linear equations in two variables simultaneously (numerically, algebraically and graphically).
2.2.5 Solve simultaneous equations with two unknowns algebraically and graphically, where both equations are linear.
2.2.6 Solve simultaneous equations with two unknowns algebraically and graphically; where the one equation is linear and the other equation is quadratic

2.3 A formal definition for the function concept is given correctly

Content:

- Function concept
- Function notation

Learning outcomes:

Learners should be able to:

2.3.1 Demonstrate the formal definition of a function

2.3.2 Use the different function notations, e.g. $f(x)=x^2+6$; $f:x\to y$

2.4 Various types of functions and relations are worked with correctly

Content:

- Sketching graphs: straight line; parabola; hyperbola; exponential and trigonometric graphs
- Characteristics of functions

Learning outcomes:

Learners should be able to:

2.4.1 Recognise relationships between variables in terms of numerical, graphical, verbal and symbolic representations and convert flexibly between these representations (tables, graphs, words and formulae).

2.4.2 Use a variety of techniques to sketch and interpret information from the following graphs of functions.

- $y=ax+q$
- $y=ax^2+q$; $y=a(x+p)^2+q$; $y=ax^2+bx+c$
- $y=\frac{a}{x}+q$; $y=\frac{a}{(x+p)}+q$
- $y=ab^x+q$; $y=ab^{x+p}+q$; where $b>0$; $b\neq 1$
- $y=a\sin kx$; $y=a\sin x+q$; $y=a\sin(x+p)$; $x\in[-360^0;360^0]$
- $y=a\cos kx$; $y=a\cos x+q$; $y=a\cos(x+p)$; $x\in[-360^0;360^0]$
- $y=a\tan kx$; $y=a\tan x+q$; $y=a\tan(x+p)$; $x\in[-360^0;360^0]$

2.4.3 Identify the following characteristics of the above functions

- Shape
- Domain (input values)
- Range (output values)
- Intercepts on the axes (where applicable)
- Asymptotes
- Axes of symmetry
- Turning points (Minima and maxima)
- Periodicity and amplitude
- Intervals in which a function increases/ decreases

2.5 Situations are recognised in which it may be helpful to manipulate representations

Content:

- Effects of parameters on graphs
- Finding equations of graphs

Learning outcomes:

Learners should be able to:

2.5.1 Investigate the effects of the parameters a and q as well as p, k , b and c on the graphs defined by:

- $y = ax + q$
- $y = ax^2 + q$; $y = a(x+p)^2 + q$; $y = ax^2 + bx + c$
- $y = \frac{a}{x} + q$; $y = \frac{a}{(x+p)} + q$
- $y = ab^x + q$; $y = ab^{x+p} + q$; where $b > 0$; $b \neq 1$
- $y = a\sin kx$; $y = a\sin x + q$; $y = a\sin(x+p)$; $x \in [-360^0; 360^0]$
- $y = a\cos kx$; $y = a\cos x + q$; $y = a\cos(x+p)$; $x \in [-360^0; 360^0]$
- $y = a\tan kx$; $y = a\tan x + q$; $y = a\tan(x+p)$; $x \in [-360^0; 360^0]$

2.5.2 Find the equations of the following types of graphs

- $y = ax + q$
- $y = ax^2 + q$; $y = a(x+p)^2 + q$; $y = ax^2 + bx + c$
- $y = \frac{a}{x} + q$; $y = \frac{a}{(x+p)} + q$
- $y = ab^x + q$; $y = ab^{x+p} + q$; where $b > 0$; $b \neq 1$
- $y = a\sin kx$; $y = a\sin x + q$; $y = a\sin(x+p)$; $x \in [-360^0; 360^0]$
- $y = a\cos kx$; $y = a\cos x + q$; $y = a\cos(x+p)$; $x \in [-360^0; 360^0]$
- $y = a\tan kx$; $y = a\tan x + q$; $y = a\tan(x+p)$; $x \in [-360^0; 360^0]$

2.6 Use a variety of techniques to sketch and interpret information for graphs of the inverse of a function

Content:

- Inverse function notation
- Sketching the inverses of functions: straight line; parabola and exponential graph
- Characteristics of the inverse of functions
- Equations of the inverse graphs

Learning outcomes:

Learners should be able to:

2.6.1 Understand the general concept of an inverse of a function

2.6.2 Use the inverse function notation correctly

2.6.3 Understand and use the relationship between a function and its inverse as reflection in the line $y = x$

2.6.4 Determine the equations of the inverses of the functions: (the general concept of the inverse of a function)

- $y = ax + q$
- $y = ax^2$
- $y = b^x$ where $b > 0$; $b \neq 1$

2.6.5 Sketch and interpret information from graphs of the inverse of the functions

- $y = ax + q$
- $y = ax^2$
- $y = b^x$ where $b > 0$; $b \neq 1$

2.6.6 Identify the following characteristics of the inverse of the functions:

$y = ax + q$; $y = ax^2$; $y = b^x$ where $b > 0$; $b \neq 1$

- Shape
- Domain (input values)
- Range (output values)
- Intercepts on the axes (where applicable)
- Asymptotes (horizontal and vertical)
- Axes of symmetry
- Turning points (Minima and maxima)
- Intervals in which a function increases / decreases

2.6.7 Determine the equations of the inverse graphs of the following functions given as a sketch:

- $y = ax + q$
- $y = ax^2$
- $y = b^x$ where $b > 0$; $b \neq 1$

2.6.8 Determine which inverses are functions and how the domain of a function may need to be restricted (in order to obtain a one-to-one function) to ensure that the inverse is a function

2.6.9 Use and understand the definition of a logarithm:

- $y = \log_b x \quad \Leftrightarrow \quad x = b^y$; where $b > 0$ *and* $b \neq 1$

2.6.10 Sketch the graph of the function define by $y = \log_b x$ for both the cases $0 < b < 1$ *and* $b > 1$

2.7 Cubic equations are solved using the factor theorem and other techniques

Content:

- Solving of cubic equations

Learning outcomes:

Learners should be able to:

2.7.1 Use and apply the remainder and factor theorem to:

> - Find the remainder
> - Prove that an expression is a factor
> - Find an unknown variable in order to make an expression a factor or to leave a remainder

2.7.2 Factorise third degree polynomials including examples that require the factor theorem and the method of inspection.
Note: No proofs are required

Topic 3: SPACE SHAPE and MEASUREMENT

Overview of SPACE SHAPE and MEASUREMENT

In this section we will study Euclidean geometry, coordinate geometry and trigonometry.

The important aspect of this topic is that it provides learners an opportunity to develop necessary concepts, skills and knowledge to describe, represent and explain properties of shapes in two-dimensional and three dimensional space with justification.

Euclidean Geometry and measurement
In this topic learners should be able to:

- Solve problems involving volume and surface area of geometric objects such cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms, right pyramids (with square, equilateral or hexagonal basis), right cones, spheres and combination of such geometrical objects.
- Investigate, conjecture and prove theorems of the geometry of circles assuming results from earlier grades and accepting that the tangent to a circle is perpendicular to the radius drawn to the point of contact.
- Solve circle geometry problems and prove riders, using circle geometry theorems, their converses and corollaries (where they exist) as well geometry results established in earlier grades.

Coordinate Geometry
In this topic learners should be able to:

- Represent geometric figures in a Cartesian co-ordinate system, and derive and apply, for any two points $(x_1;y_1)$and$(x_2;y_2)$, a formula for calculating:
  - The distance between two points
  - The gradient of the line segment joining the points
  - Conditions for parallel and perpendicular lines
  - The coordinates of the midpoints of the line segment joining the points.
- Use a two dimensional Cartesian co-ordinate system to derive and apply:

- The equation of a line through two given points
- The equation of a line through one point and parallel or perpendicular to a given line.
- The inclination of a line.
- The equation of a circle (any centre)
- The equation of tangent to a circle at a given point on the circle.

Trigonometry

In this topic learners should be able to:

- Definitions of the trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$ in right angled triangles.
- Extend the definitions of $sin\theta$, $cos\theta$and $tan\theta$ to $0° \leq \theta \leq 360°$.
- Derive and use values of trigonometric ratios for the special angles $\theta\epsilon\{0°; 30°; 60°; 90°\}$.
- Derive and use the identities $tan\theta = \frac{sin\theta}{Cos\theta}$ and $sin^2\theta + cos^2\theta = 1$.
- Derive and use the reduction formulae.
- Determine the general solution and/or specific solutions of trigonometric equations.
- Establish and apply the sine, cosine and area rules.
- Proof and use of the compound angle and double angle identities.
- Problems in 2- and 3-dimensions are solved by constructing and interpreting geometrical and trigonometric models

Prior knowledge of the following topics and their extensions are required for this topic:

Circle geometry

- Use geometry of straight lines and triangles to solve problems and to justify relationships in geometric figures.

Concepts to include are:

- angles of a triangle;
- exterior angles,
- straight lines,
- vertically opposite angles;
- corresponding angles,
- co-interior angles and
- alternate angles.

- State and apply congruency ttheorems
- Use definitions, axioms and theorems associated with quadrilateral geometry to do connected calculations and solve geometry riders.

Coordinate Geometry:

- Use the Cartesian coordinate system to plot points, lines and polygons.

Trigonometry:

- State the theorem of Pythagoras in words.
- Use the theorem of Pythagoras to calculate the missing length in a right-angled triangle leaving the answers in the most appropriate form

Details of Content coverage

3.1 Solve problems involving volume and surface area of geometric objects

Content:

- Volume and surface area of cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms.
- The effect on the volume and surface area of right prisms and cylinders, when one or more dimensions are multiplied by a constant factor.
- Volume and surface areas of spheres, right pyramids and right cones.
- Volume and surface areas of a combination of geometrical objects: right prisms, cylinders, spheres, right pyramids, and right cones.

Learning outcomes:

Learners should be able to:

3.1.1 Calculate the volume and surface area of cubes, rectangular prisms, cylinders, triangular prisms, hexagonal prisms.

3.1.2 Investigate the effect on volume and surface area of right prisms and cylinders, where one or more dimensions are multiplied by a constant factor k.

3.1.3 Calculate volume and surface areas of spheres, right pyramids and right cones.

3.1.4 Calculate volume and surface areas of a combination of the above mentioned geometrical objects.

3.1.5 Solve problems involving volume and surface area of geometrical objects.

3.2 Investigate, prove and apply the theorems of the geometry of circles:

Content:

- Circles, perpendicular lines through the centre, chords and midpoints
- Angle subtended at the centre of a circle
- Angle subtended by diameter
- Cyclic quadrilaterals
- Tangents to circles

Learning outcomes:

Learners should be able to:

3.2.1 Investigate, conjecture, prove the following theorems of the geometry of circles:

- ➢ The line drawn from the centre of the circle perpendicular to the chord bisects the chord. The perpendicular bisector a chord passes through the centre of the circle.
- ➢ The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circle (on the same side as of the chord as the centre).
- ➢ The angle subtended at the circle by a diameter is a right angle.
- ➢ Angles subtended by a chord of the circle on the same side of the chord, are equal.
- ➢ The opposite angles of a cyclic quadrilateral are supplementary.
- ➢ An exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.
- ➢ Two tangents drawn to a circle from the same point outside the circle are equal in length. The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment. (tan-chord theorem)

Note: Accept results established in earlier grades as axioms and also that a tangent to a circle is perpendicular to the radius, drawn to the point of contact.

Range: The proof of the following theorems will be examined

The line drawn from the centre of a circle to the midpoint of a chord is perpendicular to the chord.
The angle subtended by an arc at the centre of the circle is double the size of the angle subtended by the same arc at the circle (on the same side as of the chord as the centre).
The opposite angles of a cyclic quadrilateral are supplementary.
The angle between a tangent to a circle and a chord drawn from the point of contact is equal to an angle in the alternate segment

3.2.2 Use the above theorems, their converses and corollaries (where they exist) to solve and/prove riders:

- Theorems form section 3.2.1
- Equal chords subtend equal angles at the circumference
- Equal chords subtend equal angles at the centre
- Equal chords of equal circles subtend equal angles at the circumference.
- If the angle subtended by a chord at point on the circle is a right, then the chord is diameter.
- If a line is drawn perpendicular to a radius at the point where the radius meets the circle, then it is a tangent to the circle.
- If the exterior angle of a quadrilateral is equal to the interior opposite angle then the quadrilateral will be a cyclic quadrilateral.
- If two opposite angles of a quadrilateral are supplementary, then the quadrilateral is cyclic.
- If through one end of a chord of a circle, a chord is drawn making with the chord an angle equal to the angle in the alternate segment, then the line is a tangent to the circle.

3.3 Use the Cartesian coordinate system to derive and apply formulae

Content:

- Distance between any two points
- The gradient of the line segment joining two points
- Parallel and perpendicular lines
- The coordinates of the midpoints of the line segment

Learning outcomes:

Learners should be able to:

3.3.1 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the distance between any two points $(x_1;y_1)$and$(x_2;y_2)$.
3.3.2 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the gradient of the line joining any two points $(x_1;y_1)$and$(x_2;y_2)$.
3.3.3 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the gradient of a line that is parallel to another line.
3.3.4 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the gradient of a line that is perpendicular to another line.
3.3.5 Use the Cartesian co-ordinate system to derive and apply a formula to calculate the midpoint of the line segment joining any two points $(x_1;y_1)$and$(x_2;y_2)$.

3.4 Use the Cartesian coordinate system to derive and apply equations.

Content:

- The equation of a line through two given points.
- The equation of a line through one point and parallel or perpendicular to a given line.
- The inclination of a line.
- The equation of a circle (any centre).
- The equation of tangent to a circle at a given point on the circle

Learning outcomes:

Learners should be able to:

3.4.1 Use the Cartesian co-ordinate system to derive and apply the equation of a line through two given points.

3.4.2 Use a Cartesian co-ordinate system to derive and apply the equation of a line parallel or perpendicular to another given line.

3.4.3 Use a Cartesian co-ordinate system to derive and apply the angle of inclination of a line.

3.4.4 Use the Cartesian co-ordinate system to derive and apply the equation of a circle (any centre).

3.4.4 Use the Cartesian co-ordinate system to derive and apply the equation of a tangent to a circle given a point on the circle.

Note:
Straight lines to be written in the following forms only:
$y = mx + c$

$and/or \quad ax + by + c = 0\ (general\ form)$

Learners are expected to know and be able to use as an axiom "the tangent to a circle is perpendicular to the radius drawn to the point of contact."

3.5 Define, extend and apply the trigonometric ratios of $sin\theta$, $cos\theta$ and $tan\theta$

Content:

- Definitions of the trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$ using right angled triangles.
- Definitions of $sin\theta$, $cos\theta$ and $tan\theta$ for $0° \leq \theta \leq 360°$.

Learning outcomes:

Learners should be able to:

3.5.1 Define and apply the trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$, using right angled triangles.

3.5.2 Extend and apply the definitions of $sin\theta$, $cos\theta$ and $tan\theta$ for $0° \leq \theta \leq 360°$.

3.5.3 Use diagrams to determine the numerical values of trigonometric ratios for angles from 0° to 360°.

3.6 Derive and use values of trigonometric ratios for special angles

Content:

- Trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$ for special angles (without using a calculator) for $\theta\epsilon\{0°; 30°; 60°; 90°\}$.

Learning outcome:

Learners should be able to:

3.6.1 Derive and use values of trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$ for the special angles(without using a calculator) for $\theta\epsilon\{0°; 30°; 60°; 90°\}$ to simplify trigonometric expressions and solve trigonometric equations for angles in the interval $[0°; 360°]$.

3.7 Derive and use trigonometric identities

Content:

- The identities $tan\theta = \frac{sin\theta}{Cos\theta}$ and $sin^2\theta + cos^2\theta = 1$.

Learning outcome:

Learners should be able to:

3.7.1 Derive the identities $tan\theta = \frac{sin\theta}{Cos\theta}$ and $sin^2\theta + cos^2\theta = 1$

3.7.2 Use the above trigonometric identities to simplify trigonometric expressions, solve trigonometric equations for angles in the interval $[0°; 360°]$, and prove other identities.

3.8 Derive and use reduction formulae to simplify trigonometric expressions and solve trigonometric equations:

Content:

- Reduction formulae for:
- $sin(90° \pm \theta); cos(90° \pm \theta)$;
- $sin(180° \pm \theta); cos(180° \pm \theta); tan(180° \pm \theta)$;
- $sin(360° \pm \theta); cos(360° \pm \theta); tan(360° \pm \theta)$;
- $sin(-\theta); cos(-\theta); tan(-\theta)$;

Learning outcome:

Learners should be able to:

3.8.1 Derive reduction formulae for the following trigonometric expressions:

- $sin(90° \pm \theta); cos(90° \pm \theta)$;
- $sin(180° \pm \theta); cos(180° \pm \theta); tan(180° \pm \theta)$;
- $sin(360° \pm \theta); cos(360° \pm \theta); tan(360° \pm \theta)$;
- $sin(-\theta); cos(-\theta); tan(-\theta)$.

3.8.2 Use reduction formulae to:

- ➢ Reduce a trigonometric ratio of any angle to the trigonometric ratio of an acute angle.
- ➢ Simplify trigonometric expressions,
- ➢ Solve trigonometric equations for angles in the interval $[-360°; 360°]$

3.9. Determine the general solution and /or specific solutions of trigonometric equations:

Content:

- General solutions to trigonometric equations
- Specific solutions to trigonometric equations in specific intervals.

Learning outcome:

Learners should be able to:

3.9.1 Use establish trigonometric ratios $sin\theta$, $cos\theta$ and $tan\theta$, reduction formulae and established trigonometric identities to determine the general solutions of trigonometric equations.

3.9.2 Use the general solution of a trigonometric equation to determine its specific solutions in a specific interval.

3.10. Establish and apply the sine, cosine and area rules

Content:

- Sine Rule
- Cosine Rule
- Area Rule

Learning outcome:

Learners should be able to:

3.10.1 Derive and apply the sine rule.

3.10.2 Derive and apply the cosine rule.

3.10.3 Derive and apply the area rule.

3.11 Derive and apply compound angle double angle identities

Content:

- Compound angle identities
- Double angle identities

Learning outcomes:

Learners should be able to:

3.11.1 Derive the following compound angle identities

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha.\tan\beta}$$

3.11.2 Use the compound angle identities to derive the following double angle identities:

$$\sin 2\alpha = 2\sin\alpha\ \cos\ \alpha$$
$$\cos\ 2\alpha = \begin{cases} \cos^2\alpha - \sin^2\alpha \\ 2\cos^2\alpha - 1 \\ 1 - 2\sin^2\alpha \end{cases}$$

3.11.3 Determine the specific solutions of trigonometric expressions using compound and double angle identities without a calculator. (e.g. sin 120⁰, cos 75⁰ etc.)

3.11.4 Use compound angle identities to simplify trigonometric expressions and to prove trigonometric identities.

3.11.5 Determine the specific solutions of trigonometric equations by using knowledge of compound angles and identities.

Note:

- Solutions: $[0;360°]$
- Identities limited to:

  $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

  $\sin^2\theta + \cos^2\theta = 1$
- Double and compound angle identities are included

Note: radians are excluded

3.12 Solve problems in 2- and 3-dimensions by constructing and interpreting geometrical and trigonometric models

Content:

- Two dimensional problems
- Three dimensional problems

Learning outcomes:

Learners should be able to:

3.12.1 Solve problems in two dimensions by using the trigonometric functions ($sin\theta$, $cos\theta$ and $tan\theta$)in right –angled triangles and by constructing and interpreting geometric and trigonometric models.

3.12.2 Solve problems in two dimensions by using the sine, cosine and area rules, and by constructing and interpreting geometric and trigonometric models.

3.12.3 Solve problems in three dimensions by constructing and interpreting geometric and trigonometric models. Note: compound angle identities are excluded

Topic 4: DATA HANDLING AND PROBABILITY

Overview of DATA HANDLING and PROBABILITY

In this section the learner should be able to collect, organize, analyse and interpret data to establish statistical and probability methods to solve related problems.

A fundamental aspect of this outcome is that it provides opportunities for learners to be become aware and also participate in the completion of the 'statistical cycle' of formulating questions, collecting appropriate data, analysing and representing this data, and so arriving at conclusions about the questions raised.

In this topic learners should be able to:

- Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical ungrouped data
- Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical grouped data
- Understand and demonstrate the use of variance and standard deviations as measures of dispersion of a set of bivariate data
- Understand the various concepts and representations relating to data handling and its uses, and used them correctly

Prior knowledge of the following topics and their extensions are required for this topic:

Data handling

Representation

- Bar and compound bar graphs
- Histograms (grouped data)
- Frequency polygons
- Pie charts
- Line and broken line graphs

Details of Content coverage

4.1 Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical ungrouped data

Content:

- Measures of central tendency

- Five number summary
- Box and whisker diagrams
- Measures of dispersions

Learning outcomes:

Learners should be able to:

4.1.1 Calculate the:

- Mean
- Median
- Mode

4.1.2 Work out the five number summary by:

- Calculating the maximum, minimum and quartiles
- Determine the fences
- Constructing the box and whisker diagram
- Indicating any outliers

4.1.3 Interpret the meaning of the representation of the box and whisker diagram with its outliers

4.2 Calculate, represent and interpret measures of central tendency and dispersion in univariate numerical grouped data

Content:

- Frequency distribution table
- Ogive curve
- Quartile values
- Histograms
- Mean, median and mode

Learning outcomes:

Learners should be able to:

4.2.1 Construct a frequency distribution table by grouping data into classes

4.2.2 Calculate the cumulative frequency and plot the Ogive curve

4.2.3 Use the Ogive curve to estimate quartile values

4.2.4 Construct histograms using tabulated grouped data

4.2.5 Calculate the using the formulae:

$$\bar{x} = \frac{\sum f_i x_i}{n}$$

$$Me = l + \frac{\left(\frac{n}{2} - F\right)}{f} \times c$$

$$Mo = l + \frac{f_m - f_{m-1}}{2f_m - f_{m-1} - f_{m+1}} \times c$$

4.3 The use of variance and standard deviations as measures of dispersion of bivariate data is understood and demonstrated

Content:

- Variance and standard deviation of bivariate data

Learning outcomes:

Learners should be able to:

4.3.1 Calculate:

> - Variance and
> - Standard deviation manually for small sets of bivariate data only

4.3.2 Interpret the meaning of variance and standard deviation for small sets of data only

4.4 The various concepts and representations relating to data handling and data and its uses are understood and used correctly

Content:

- Scatter plots
- Correlation
- Best fit function
- Regression lines

Learning outcomes:

Learners should be able to:

4.4.1 Represent bivariate numerical data as a scatter plot

4.3.2 Identify intuitively whether a linear, quadratic or exponential function would best fit the data

4.4.3 Draw the intuitive line of best fit.

Range:

Data given should include problems relate to health, social, economic, cultural, political and environmental issues

For small sets of data only (limited to 8)

4.4.4 Use least square regression method to determine a function which best fits a given set of bivariate data

4.4.5 Use a calculator to calculate the linear regression line which best fist a given set of numerical data

4.4.6 Use the regression line to predict the outcome of a given problem

4.4.7 Use a calculator to calculate the correlation coefficient of a set of bivariate numerical data and make relevant deductions

SECTION 4

ASSESSMENT IN MATHEMATICS

Specifications for External Assessment in Mathematics – NQF Level 4.

A national examination is conducted annually in October or November each year by means of two three hour examination papers (Paper 1 and Paper 2) set externally and marked and moderated externally.

4.1 Content, sub-topics & weightings

The content topics and its weightings examined in each of Paper 1 and Paper 2 are indicated in Table 1.

Description	Duration	Weightings (Marks)
PAPER 1	**3h**	
Patterns, Sequences and Series		45 ± 3
Finance, growth and decay		35 ± 3
Functions, Graphs and Algebra		70 ± 3
TOTAL		**150**
PAPER 2:	**3h**	
Data handling		30 ± 3
Circle geometry & measurement		40 ± 3
Co-ordinate geometry		30 ± 3
Trigonometry		50 ± 3
TOTAL		**150**

Table 1

4.2 Weighting of levels of Cognitive demand

The four cognitive levels used to guide the setting of the external examination papers are:

- Knowledge;
- Routine procedures;
- Complex Procedures and
- Problem Solving.

The suggested weightings for Paper 1 and Paper 2 are shown in Table 2.

Knowledge	Routine Procedures	Complex procedures	Problem Solving
20%	40%	25%	15%

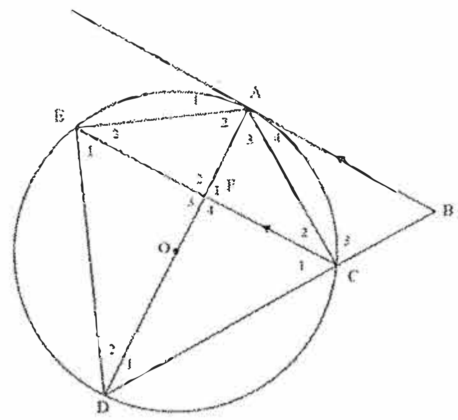
Table 2

4.3 Descriptors for each cognitive level with illustrative examples

The four cognitive levels used to guide all assessment tasks are based on those suggested in the TIMSS study of 1999. Descriptors for each level and the approximate percentages of tasks, tests and examinations which should be at each level are given below:

COGNITIVE LEVELS	EXPLANATION OF SKILLS TO BE DEMONSTRATED	EXAMPLES
KNOWLEDGE (K) **20%**	• Theorems • Straight recall • Identification of correct formula on the Formulae sheet (no changing of the subject) • Use of simple mathematical facts • Know and use of appropriate mathematical vocabulary • Knowledge and use of formulae (without changing of the subject) • Answer only: e.g. Definitions *All of the above will be based on known knowledge.*	1. Write down the domain and range of the function f where $f(x) = -x^2$ 2. The angle $A\hat{O}B$ subtended by arc AB at the centre O of a circle is equal to .... 3. Write down the first five terms of the sequence with general term $T_k = \frac{1}{3k-1}$ 4. Find the asymptotes, axis of symmetry, x- and y intercepts: reading from the graph. 3 Simplify: $\sin x \cos y + \cos x \sin y$
ROUTINE PROCEDURES (R) **40%**	• Algorithms • Estimation and appropriate rounding of numbers • Application of prescribed theorems • Identification and use of correct formula on the Formulae sheet and manipulating of formulae (after changing of the subject) • Problems can involve the integration of different LOs • Perform well-known procedures • Simple applications and calculations which might involve a few steps • Derivation and interpretation from given information may be involved • Generally similar to those encountered in class	1. Solve for x : $x^3 + 8x^2 + 17x + 10 = 0$ 2. Given: $f(x) = 4x - 1$ Determine the inverse function f^{-1} 3. Sketch the graphs of $f(x) = 4x - 1$, f^{-1} and $y = x$ line on the same set of axes. What do you notice? 4. Find the 8th term in the series 3; 5; 7;.....

	All of the above will be based on known procedures.	5. Determine the largest value of *n* such that $\sum_{i=1}^{n}(3i-2) < 2000$ 6. Determine the value of $Cos\ 75°$, without using a calculator 7. O is the centre of the circle and B is the midpoint of the chord AC. Prove using Euclidean geometry methods, the theorem that states OB is perpendicular to AC. (NB. The diagram will be provided) 8. Find the value of $\sin 3x$ in terms of $sinx$.
COMPLEX MULTI-STEP PROCEDURES (C) **25%**	• Problems are mainly unfamiliar and learners are expected to solve by integrating different LOs • Problems involve complex calculations and/or higher order reasoning • Problems do not always have an obvious route to the solution but involve: ➢ using higher level calculation skills and reasoning to solve problems ➢ mathematical reasoning processes • These problems are not necessarily based on real world contexts and may be abstract requiring fairly complex procedures in finding the solutions • Could involve making significant connections between different representations • Require conceptual understanding	1. Find the sample regression equation by using the method of least squares. 2. Drawing a box and whisker diagram when the outliers are outside the upper and or lower fence or both 3. If $f(x) = -x^2 + 1$; Determine the equation of the inverse of $f(x) = -x^2 + 1;\ x \geq 0$ 4. Sketching of graphs with horizontal or vertical shift. 5. Determine the 5th term of the geometric sequence of which the 8th term is 6 and the 12th term is 14. 7. Prove that $\frac{1+\sin 2x}{\cos 2x} = \frac{\cos x + \sin x}{\cos x - \sin x}$
SOLVING PROBLEMS	• Non-routine problems (which are not necessarily difficult) • Using higher level cognitive skills, reasoning and processes to solve non-	1. If $a(x) = x^5 - 2x^3 + px - 1$ is divided by $x-1$, the

(P) **15%**	routine problems • Being able to break down a problem into its constituent parts – identifying what is required to be solved and then using appropriate methods in solving the problem	remainder is $-\frac{1}{2}$. Determine the value of p. 2. In the figure below, AB is a tangent to the circle with centre O. AC = AO and BA || CE. DC produced, cuts tangent BA at B. Prove that AD = 4AF.  3. Jeffry invests R700 per month into an account earning interest at a rate of 8% per annum, compounded monthly. His friend also invests R700 per month and earns interest compounded half yearly at r% per anum. Jeffrey and his friends' investments are worth the same at the end of 12 months. Calculate r.

4.4 Moderation of assessments

Moderation refers to the process that ensures that the assessment tasks are fair, valid and reliable. Moderation should be implemented at school, district, provincial and national levels. Comprehensive and appropriate moderation practices must be in place to ensure quality assurance for all subject assessments.

The ANALYSIS GRID in table 3 will be used to moderate papers 1 and 2:

Table 3

LEVEL:	EXAMINER:
TASK: External Examination – PAPER 1 & 2	MODERATOR:

Topic(s)	AC(s)	LO(s)	Question No.	Mark allocation	Time allocation (± Min) (*L2: Mark x 1,8 =≈*)	Cognitive Level (Mark allocation)				Difficulty (Mark allocation)				Total
						Know-ledge (K) 20%	Comprehension & Application (C) 60%		Analysis (A) 20%	Level 1	Level 2	Level 3	Level 4	
						1 Knowing	2 Routine procedures	3 Complex Multi-step procedures	4 Reasoning and reflecting	Easy (K & RE)	Moderate (RM)	Difficult (RD & C)	Problem solving (P)	
						20 %	40 %	25 %	15 %					
1. Numbers	1.1	1.1.1	2.1	3	5	3				3				3

TOTAL						20 %	40 %	25 %	15 %					

Curriculum Coverage

Description	Duration	Suggested Weightings (Marks)	Actual weightings
PAPER 1	3h		
Patterns, Sequences and Series		45± 3	
Finance, growth and decay		35 ± 3	
Functions, Graphs and Algebra		70 ± 3	
TOTAL		**150**	
PAPER 2:	**3h**		
Data handling		30 ± 3	
Circle geometry		40 ± 3	
Co-ordinate geometry		30 ± 3	
Trigonometry		50 ± 3	
TOTAL		**150**	

GLOSSARY

Amplitude – the distance from the x-axis to the maximum value.

Angle of depression – the angle below the horizontal line.

Angle of elevation - the angle above the horizontal line.

Angle of inclination – the angle that a line makes with positive direction of the x-axis.

Arc – a part of the **circumference** of a **circle.**

Asymptote – a straight line to which a curve continuously draws nearer without ever touching it.

Axis of symmetry – the line about which the function is symmetrical.

Axis of symmetry of a graph– a line over which a graph can be reflected so that one half of the graph fits exactly over the other half, for example the lines y=x and y=-x are the axes of symmetry of the hyperbola graph xy=k.

Binomial – an expression with two terms (for example, $x^3 - 5x^2$).

Bivariate data- two sets of data values that both vary.

Bonds- Written promise by a company that borrows money usually with fixed interest payment until maturity (repayment time)

Box-and whisker diagram - diagram that represents the spread of data, with the 'box' representing the interquartile range and the 'whiskers' showing minimum and maximum values.

Cartesian plane – a 2D-plane (formed by a horizontal and a vertical number line that intersect at 0 on each).

Centre of circle – A point about which all the points on the **circumference** of a circle are **symmetrical.**

Chord – a straight line segment joining any two points on a **circumference** of a **circle.**

Circle – a curve that is the **locus** of a point which moves at a fixed distance from a fixed point (centre).

Circumference – a perimeter of a circle or any closed curve.

Collinear- points that lie on the same line; the gradients between any two of these points will be the same.

Common difference- If the differences between consecutive terms of a sequence are all equal then these are collectively known as the common difference.

Common factor – A factor that is shared by two or more numbers or expressions

Common ratio- If the ratios between consecutive terms of a sequence are all equal then these are collectively known as the common ratio.

Compounding- Calculating the interest periodically over the life of the loan and adding it to the principal.

Compound interest- The interest that is calculated periodically and then added to the principal. The next period the interest is calculated on the adjusted principal (old principal plus interest).

Conjecture- a tentative solution inferred from collected data.

Concentric- circles that share the same centre.

Congruent- identical in all respects.

Converse –reversing the logic and proving a theorem in reverse.

Co-ordinates – a pair of numbers to indicate appoint on a graph.

Corresponding angles – angles in similar positions; angles which lie on the same side of a pair of **parallel lines** and the **transversal** (see **transversal**).

Corollary- a deduction based on the result of a theorem.

Cumulative frequency – the running total of frequecnies.

Cyclic quadrilateral-a four sided figure with all four points lying on the circumference of a circle.

Diagonal – a straight line joining opposite vertices.

Diameter – any straight line drawn from any point on a circumference of a circle, through the centre to a second point on the circumference.

Dependent variable – the element of the **range** of a function which depends on the corresponding value(s) of the domain (e.g. in $y = f(x) = \pi x^2$ the area of a circle (y) depends on the radius, x if x is the **independent variable** and y the dependent variable.

Depreciation- The process of allocating the cost of an asset (less residual value) over the asset's estimated life.

Enlargement – a mapping that increases the distances between parallel lines by the same factor in all directions.

Effective rate- True rate of interest. The more frequent the compounding, the higher the effective rate.

Exterior angle - outside angle of a polygon formed by one of the sides which has been extended.

Factorise – to write a number as the product of its factors

Future value (FV)- Final amount of the loan or investment at the end of the last period. Also called *compound amount.*

General solution- the formula which lists all possible solutions to a trigonometric equation; takes into account the period of the trigonometric equation so an angle can be positive or negative

Gradient – in general a slope i.e. an inclination to the horizontal expressed either as an angle the path makes with the horizontal or as the ratio between the vertical distance travelled and the horizontal distance travelled.

Horizontal asymptote – a y-value the graph approaches, but never reaches or crosses.

Horizontal axis – the horizontal line of the Cartesian plane (also called the x-axis).

Hypotenuse – the side opposite the right angle in a right angled triangle.

Histogram – a graph that uses bars or columns to represent data values that have been grouped into class intervals. The width of a bar represents the class interval and the height/length of the bar represents the number or frequency of values in that class.

Independent variable – as used in dealing with functions: the value that determines the value of the **dependent variable** (e.g. in $y = f(x) = \pi x^2$ the area of a circle (y) depends on the radius, (x) if x is the independent variable and y the dependent variable.

Interquartile range – the difference between the third and first quartiles.

Linear graph – a graph in which all points lie on a straight line.

Line segment - a portion of a straight line between two points; it has a finite length

Logarithm- A quantity representing the power to which a fixed number (the base) must be raised to produce a given number.

Lower quartile – one quarter of the data in a sample has a value less than or equal to the value of the lower quartile. Three quarters of the data in a sample has a value less than or equal to the value of the upper quartile. The median, upper quartile and lower quartile divide the data into four equal parts.

Measures of central tendency – values that tell you where the middle (or centre) of the data lies.

Measures of dispersion – values that show how spread out (or grouped) the data values are.

Measures of spread – numerical values that summarise how spread out the data values are. Examples are range, interquartile range and semi-interquartile range.

Median- the middle value after data is arranged in ascending order (or descending order); the median divides ordered data into two halves; each having the same number of data items.

Midpoint – the point in the middle of a line segment.

Mode- the most common value.

Normal – a line through a given point on a curve **perpendicular** to the **tangent** to the curve at that point.

Outstanding balance on a loan- Present value of a series a of payments still to be made.

Outlier – a value of measurement that is abnormally higher or lower than the rest of the data.

Parallel Lines – lines that are always the same distance apart and lie in the same direction

Parameter – a constant whose value determines in part how interrelated variables are expressed and through which they may then be regarded as being **dependent** upon one another.

Periods- Number of years times the number of times compounded per year.

Perpendicular – at a right angles or at 90°; lines are perpendicular when the angles between the lines is90°.

Perpendicular bisector – a line which is drawn at a right angles to a **line segment** and divides it in halve

Present value annuity- Amount of money needed today to receive a specified stream (annuity) of money in the future.

Present value (PV)- How much money will have to be deposited today (or some date) to reach a specific amount of maturity (in the future).

Radius – the distance from a centre of a circle to any point on the **circumference** or the distance from the centre of a **sphere** to any point on its surface

Range of a graph – the set of all the output values is called the range of a graph

Rate of interest- Percent of interest that is used to commute the interest charge on a loan for a specific time.

Reduction – a mapping that reduces the distances between parallel lines by the same factor in all directions.

Reduction formulae – trigonometric identities that express the trigonometric ratios of an angle of any size in terms of the trigonometric ratios of an acute angle.

Reflection – when the graph *flips* across a line (usually the x-or y-axis or the line y=x).

Regression line- the 'line of best fit' for a set of plotted points; also called the line of least squares.
Secant – a line that cuts a given curve.

Sector – a part of a circle lying between two radii (singular is radius) and either of the arcs that they cut off.

Sigma – the symbol $\sum$ denoting the sum.

Simple interest - Interest is only calculated on the principal. In $I = P \times R \times T$, The interest plus original principal equals the maturity value of an interest-bearing note.

Simple interest formula - Interest= Principal x Rate x Time.

Specific solutions- solutions which satisfy a given trigonometric equation in a restricted interval

***Stretch** – the shape of a graph or object* undergoes a vertical or horizontal increase or decrease in scaling.

Supplementary angles – two angles that add up to 180°; each angle is said to be the supplement of the other.

Symmetrical – a geometric figure is symmetric if it is identical with its own reflection in an axis of symmetry (a line through the centre of the figure).

Tangent – a line that touches a curve at only one point.

Time- Expressed as years or fractional years, used to calculate the simple interest.

Theorem- A formal proof of a geometric statement.

Transformation (1) – the change of one figure (transformation geometry) or one expression (algebra) to another.

Transformation (2) – a change in the position or size of an object or expression.

Translation – a **transformation** that moves or points the same distance in a common direction

Transversal – any line that intersects two or more lines.

Trigonometric equation – an equation involving trigonometric ratios which is true only for certain values of the unknown variable.

Trigonometric identity- an identity which is true for all values of an unknown variable, for which both sides of the identity are defined (so no zero denominators).

Turning point – a maximum or minimum point on a curve where the y-value changes from increasing to decreasing or vice versa (and the tangent is horizontal).

Ungrouped data- raw data that have been collected and the specific values are given.

Univariate data- data that involve one variable and do not deal with causes or relationships.

Upper quartile – the $\frac{3}{4}(n+1)th$ data value; Q_3.

Variance – the average of the squared differences from the mean.

Vertical axis – the vertical line perpendicular to the horizontal axis (also called the y-axis).

Vertex- the point at which two straight lines meet to form an angle.

x-intercept (root) – the point where the graph crosses the x-axis (where $y = 0$).

y-intercept – the point where the graph crosses the y-axis (where $x = 0$).

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